Obtaining System Performance Measures

From Routh's Algorithm

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Diamessis' approach to the calculation of system performance measures [1] was quite interesting. His state matrix equation is based on Schwarz' form [2] which is

$$\dot{x} = A x \qquad \text{N } 66\text{-}81055 \qquad (1)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots\dots\dots & 0 & 0 \\ -a_1 & 0 & 1 & \dots\dots\dots & 0 & 0 \\ 0 & -a_2 & 0 & \dots\dots\dots & 0 & 0 \\ & \dots\dots\dots\dots & & 1 & 0 & 0 \\ & \dots\dots\dots\dots & & 0 & 1 & 0 \\ 0 & \dots\dots\dots & -a_{n-2} & 0 & 0 \\ 0 & \dots\dots\dots\dots & & -a_{n-1} & -a_n \end{bmatrix} \qquad (2)$$

Kalman, Bertram's Liapunov function [3] is an important application of Schwarz form. However, the evaluation of a_i from an arbitrary form is not direct and simple. Schwarz used many linear transformations to obtain a set of a_i. Parks [3] found a_i from Hurwitz determinants. The formula they obtained are not easy to use. This note attempts to find a_i elements from Routh's algorithm only.

The characteristic equation of (1) is

$$|\lambda I - A| = \lambda^n + \mu_1 \lambda^{n-1} + \mu_2 \lambda^{n-2} + \dots + \mu_{n-1} \lambda + (-1)^n |A| \qquad (3)$$

If the system is fifth order, for example, we have

$$|\lambda I - A| = \lambda^5 + a_5 \lambda^4 + (a_1 + a_2 + a_3 + a_4) \lambda^3 +$$

$$(a_1 + a_2 + a_3) a_5 \lambda^2 + (a_1 a_3 + a_1 a_4 + a_2 a_4) \lambda$$

$$+ a_1 a_3 a_5 \tag{3a}$$

The Routh array of (3a) is as follows:

$$C_o = 1 \qquad (a_1 + a_2 + a_3 + a_4) \qquad (a_1 a_3 + a_1 a_4 + a_2 a_4)$$

$$C_1 = a_5 \qquad (a_1 + a_2 + a_3) a_5 \qquad a_1 a_3 a_5$$

$$C_2 = a_4 \qquad (a_1 + a_2) a_4$$

$$C_3 = a_3 a_5 \qquad a_1 a_3 a_5$$

$$C_4 = a_2 a_4$$

$$C_5 = a_1 a_3 a_5$$

where C_i are the new symbols of the first column of Routh array, and a_i can be evaluated in terms of them or

$$a_5 = C_1$$

$$a_4 = C_2$$

$$a_3 = \frac{C_3}{C_1}$$

$$a_2 = \frac{C_4}{C_2} \tag{4}$$

$$a_1 = \frac{C_5}{C_3}$$

Parks did not obtain these simple relations but used the combinations of Hurwitz determinants. His corresponding formula is so complicated and long that it is difficult to follow.

REFERENCES

1. Diamessis, J. E., "A New Method for Calculating System Performance Measures". IEEE Proceedings, p. 1240, October 1964.

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3. Kalman, R. E., and Bertram, J. E., "Control System Analysis and Design Via the Second Method of Liapunov". Trans. ASME, Series D, pp. 371-393, June 1960.